BS-136A	Calculus and Ordinary Differential Equations						
L	T	P	Credit	Major Test	Minor Test	Total	Time
3	1	-	4	75	25	100	3 h
Purpose	To familiarize the prospective engineers with techniques inmultivariate integration, ordinary and partial differential equations and complex variables.						
Course Outcomes							
CO1	To introduce effective mathematical tools for the solutions of differential equations that model physical processes.						
CO 2	To acquaint the student with mathematical tools needed in evaluating multiple integrals and their usage.						
CO 3	To introduce the tools of differentiation of functions of complex variable that are used in various techniques dealing engineering problems.						
CO 4	To study various fundamental concepts of Vector Calculus.						
CO 5	To introduce the tools of integration of functions of complex variable that are used in various techniques dealing engineering problems.						
CO 6	To provide the conceptual knowledge of Engineering mathematics.						

UNIT-I (10 hrs)

First order ordinary differential equations: Exact, linear and Bernoulli's equations, Euler's equations, Equations not of first degree: equations solvable for p, equations solvable for y, equations solvable for x and Clairaut's type.

Ordinary differential equations of higher orders:

Second order linear differential equations with constant coefficients, method of variation of parameters, Cauchy and Legendre's linear differential equations.

UNIT-II (10 hrs)

Multivariable Calculus (Integration): Multiple Integration: Double integrals (Cartesian), change of order of integration in double integrals, Change of variables (Cartesian to polar)

Applications: areas and volumes; Triple integrals (Cartesian), orthogonal curvilinear coordinates, Simple applications involving cubes, sphere and rectangular parallelepipeds.

UNIT-III (10hrs)

Vector Calculus: Introduction, Scalar and Vector point functions, Gradient, divergence & Curl and their properties, Directional derivative.

Line integrals, surface integrals, volume integrals, Theorems of Green, Gauss and Stokes (without proof).

UNIT-IV (10 hrs)

Complex Variable – Differentiation: Differentiation, Cauchy-Riemann equations, analytic functions, harmonic functions, findingharmonic conjugate; elementary analytic functions (exponential, trigonometric, logarithm) andtheir properties;

Complex Variable – Integration:Contour integrals, Cauchy-Goursat theorem (without proof), Cauchy Integral formula (withoutproof), Taylor's series,zeros of analytic functions, singularities, Laurent's series; Residues, Cauchy Residue theorem (without proof).

Suggested Books:

1. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9th Edition, Pearson,Reprint, 2002.
2. Erwin kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.

3. Erwin kreyszig and SanjeevAhuja, Applied Mathematics- II, Wiley India Publication, 2015.
4. W. E. Boyce and R. C. DiPrima, Elementary Differential Equations and Boundary ValueProblems, 9th Edn., Wiley India, 2009.
5. S. L. Ross, Differential Equations, 3rd Ed., Wiley India, 1984.
6. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice HallIndia, 1995.
7. E. L. Ince, Ordinary Differential Equations, Dover Publications, 1958.
8. J. W. Brown and R. V. Churchill, Complex Variables and Applications, 7th Ed., Mc-Graw Hill,2004.
9. N.P. Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications, Reprint, 2008.
10. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010.
Note: The paper setter will set the paper as per the question paper templates provided.

LECTURE PLAN

Subject Name	Calculus and Ordinary Differential Equations
Subject Code	BS-136A
Course:-	**B.Tech. (ME)**
Semester:-	**2nd Semester**

Lecture	Topic
L1	Exact Differential Equation
L2	do
L3	linear and Bernoulli’s equations
L4	Euler’s equations
L5	do
L6	Equations solvable for p,equations solvable for y, equations solvable for x
L7	do
L8	Clairaut’s type
L9	Second order linear differential equations with constant coefficients
L10	Method of variation of parameters
L11	Cauchy and Legendre’s linear differential equations
L12	do
L13	Double integrals (Cartesian)
L14	Change of order of integration in double integrals
L15	Change of variables (Cartesian to polar)
L16	Areas and Volumes
L17	Triple integrals (Cartesian)
L18	Orthogonal curvilinear coordinates
L19	Simple applications involving cubes
L20	do
L21	Sphere and rectangular parallelepipeds
L22	do
L23	Scalar and Vector point functions,
L24	Gradient and its properties, Directional derivative

L25	do
L26	divergence & Curl and their properties
L27	do
L28	Line integrals, surface integrals, volume integrals
L29	do
L30	Green Theorem
L31	Gauss Theorem
L32	Stokes Theorem
L33	Differentiation, Cauchy-Riemann equations
L34	do
L35	Analytic functions, harmonic functions, finding harmonic conjugate
L36	do
L37	Elementary analytic functions (exponential, trigonometric, logarithm) and their properties
L38	do
L39	do
L40	Contour integrals,
L41	Cauchy-Goursat theorem (without proof)
L42	Cauchy Integral formula (withoutproof)
L43	Taylor's series
L44	Zeros of analytic functions
L45	Singularities
L46	Laurent's series; Residues, Cauchy Residue theorem (without proof)
L47	do
L48	Revision

Tutorial Sheet-1

Q.1 Solve $y\left(xy+2x^2y^2\right)dx + x\left(xy - x^2y^2\right)dy = 0$

Q.2 Solve $\left(D^4+2D^2+1\right)y = x^2\cos x$

Q.3 Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

Q.4 Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$

Q.5 Solve $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$

Tutorial Sheet-2

Q.1 Calculate $\iint r^3 dr d\theta$ over the area include between the circles r = 2 sin ɵ & r =4sin ɵ.

Q.2 Evaluate the following integrals. $(i)\quad \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$

$(ii)\quad \iint xydxdy$ over the positive quadrant of the Circle $x^2+y^2=a^2$

Q.3 Calculate the area included between the curve r = a (sec ɵ + cos ɵ) and its asymptote.

Q.4 Evaluate the following Integrals. $(i)\quad \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$

$$(ii)\quad \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$$

Q.5 Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes y+z=4 and z = 0

Q.6 Evaluated $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates.

Tutorial Sheet-3

Q.1 In what direction from (3,1,-2) is the directional derivative of ɵ = $x^2y^2z^4$ is maximum? Find also the magnitude of this maximum.

Q.2 Find div F and curl F where F = grad $(x^3+y^3+z^3-3xyz)$.

Q.3 Find the value of a if the vector $(ax^2y + yz)I + (xy^2 - xz^2)J + (2xyz- 2x^2y^2)K$ has zero divergence. Find the curl of the above vector which has zero divergence.

Q.4 Show that $\nabla.(\phi\nabla\psi - \psi\nabla\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$

Q.5 Verify Green's theorem for $\int_C \left[(xy + y^2)dx + x^2dy\right]$ where C is bounded by y = x and y = x^2.

Q.6 Verify Stoke's theorem for the vector field F = (2x-y)I $-yz^2J-y^2zk$ over the upper half surface of $x^2+y^2+z^2 = 1$ bounded by its projection on the xy- plane.

Q.7 Verify divergence theorem for F = $(x^2-yz)I+(y^2-zx)J+(z^2-xy)K$ takes over the rectangular parallelopiped $0\le x\le a$, $0\le y\le b$, $0\le z\le c$.

Tutorial Sheet-4

Q.1 Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied at that point.

Q.2 If f(z) is a regular function of z . Prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

Q.3 Find the regular function whose imaginary part is $e^{-x}(x\sin y - y\cos y)$

Q.4 Show that the polar form of Cauchy-Riemann equations

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad , \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$. Deduce that $$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Sample Paper

BT-II

Calculus and Ordinary Differential Equations

Paper: BS-136A

Time: Three Hours Maximum marks: 75

Note: The Examiners will set eight questions, taking two from each unit. The students are required to attempt five questions in all selecting at least one from each unit. All questions will carry equal marks.

UNIT-I

Q.1 (a) Solve $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$

(b) Solve $p^3 - p(x^2 + xy + y^2) + xy(x+y) = 0$

Q.2 (a) Solve $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$

(b) Solve by the Method of Variation of parameters:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$$

UNIT-II

Q.3(a) Calculate $\iint r^3 dr d\theta$ over the area include between the circles r = 2 sin ɵ & r =4sin ɵ.

(b) Evaluate $\int_0^{\frac{a}{2}}\int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2)\,dxdy \quad (a>0)$

Q.4 (a) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes y+z=4 and z = 0.

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}}$ by changing to spherical coordinates.

UNIT-III

Q.5 (a) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

(b) Find Interpretation of divergence.

Q.6(a) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$ where C is bounded by y = x and y = x^2.

(b) Verify Stoke's theorem for the vector field F = (2x-y)I –yz^2J-y^2zk over the upper half surface of x^2+y^2+z^2 = 1 bounded by its projection on the xy- plane.

UNIT-IV

Q.7(a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied at that point.

(b) If f(z) is a regular function of z . Prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

Q.8(a) Find the regular function whose imaginary part is $e^{-x}(x \sin y - y \cos y)$

(b) Show that the polar form of Cauchy-Riemann equations

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$